[5] Hegglin, B., "Dynamic buckling of columns," Stanford Univ., SUDAER Rept. 129 (June 1962).
[6] Timoshenko, S. P. and Gere, J. M., *Theory of Elastic Stability* (McGraw-Hill Book Co., Inc., New York, 1961), 2nd ed.
[7] Lundergan, C. D. and Herrmann, W., "Equation of state of 6061-T6 aluminum at low pressures," J. Appl. Phys. **34**, 2046–2052 (1963).
[8] Maiden, C. J., Gehring, J. W., and McMillan, A. R., "Investigation of fundamental mechanism of damage to thin targets by hypervelocity projectiles," General Motors Defense Research Labs., TR 63-208 (March 1963).
[9] Altshuler, L. V., Krupnikov, K. K., Ledener, B. N., Zhuchiklin, V. I., and Braznik, M. I., "Dynamic compressibility and equation of state of iron," Soviet Phys.-JETP **34**, 874—885 (1958).

Rapid Technique for Calculating Times of Maximum Thermal Stress in Simple Shapes

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Nomenclature

a = inside radius
b = outside radius
c_p = specific heat
E = modulus of elasticity
$\dot{Q}$ = heat flux
$\dot{q}$ = total heat absorption
r = radius
T = temperature
t = time
V = volume
α = coefficient of thermal expansion
μ = indicates surface condition
ν = Poisson's ratio
ρ = density
σ_θ = hoop stress

A METHOD has been developed which permits rapid calculation of thermal stress in uniformly heated cylinders, spheres, or plates. In addition, the technique provides a means for determining the time at which maximum thermal stresses occur. Thus, a good deal of laborious calculation can be avoided.

Circumferential thermal stress in a hollow circular cylinder, heated uniformly, can be computed[1] from

$$\sigma_\theta = \left(\frac{E\alpha}{1-\nu}\right)\frac{1}{r^2}\left\{\frac{r^2+a^2}{b^2-a^2}\int_a^b rTdr + \int_a^r rTdr - r^2T\right\} \tag{1}$$

However, in most practical problems, the maximum stresses occur at one of the surfaces, where

$$\sigma_{\theta_a} = \frac{E\alpha}{1-\nu}\left\{\frac{2}{b^2-a^2}\int_a^b rTdr - T_a\right\}$$
$$\sigma_{\theta_b} = \frac{E\alpha}{1-\nu}\left\{\frac{2}{b^2-a^2}\int_a^b rTdr - T_b\right\} \tag{2}$$

But, the net rate of heat absorption in any volume is

$$\dot{q} = \int_V \rho c_p \frac{\partial T}{\partial t}\, dV \tag{3}$$

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Or for a unit surface area, heated uniformly at the exterior by a flux Q_b and insulated at the interior

$$\dot{Q}_b = \int_a^b \rho c_p \frac{\partial T}{\partial t}\left(\frac{r}{b}\right) dr \tag{4}$$

Integrating over time, and substituting into Eq. (2),

$$\sigma_{\theta_a} = \sigma_{\theta_a}(0) + \frac{E\alpha}{1-\nu}\left\{\left(\frac{2b}{b^2-a^2}\right)\frac{1}{\rho c_p}\times \int_0^t \dot{Q}_b dt + T_a(0) - T_a(t)\right\}$$
$$\sigma_{\theta_b} = \sigma_{\theta_b}(0) + \frac{E\alpha}{1-\nu}\left\{\left(\frac{2b}{b^2-a^2}\right)\frac{1}{\rho c_p}\times \int_0^t \dot{Q}_b dt + T_b(0) - T_b(t)\right\} \tag{5}$$

where $\sigma_\theta(0)$ is the initial, steady-state stress (i.e., at $t = 0$). The same approach can be taken for a sphere and a plate restrained in bending. The general result is

$$\frac{\sigma_\mu(t) - \sigma_\mu(0)}{E\alpha/(1-\nu)} = \left\{\frac{nb^{n-1}}{b^n - a^n}\frac{1}{\rho c_p}\times \int_0^t \dot{Q}dt + T_\mu(0) - T_\mu(t)\right\} \tag{6}$$

where $n = 3, 2,$ or 1 for a sphere, cylinder, or plate.

Thus it is seen that the thermal stress at the surfaces of a uniformly heated sphere, cylinder, or plate can be determined from the thermal history by one simple integration. Also, the time of maximum thermal stress can be simply obtained by maximizing Eq. (6) with respect to time. Then, in the general case, the time of maximum stress is determined from the thermal history simply by finding the time at which

$$\dot{Q}_b = \left(\frac{b^n - a^n}{nb^{n-1}}\right)\rho c_p \frac{dT_\mu}{dt} \tag{7}$$

It should be emphasized that the foregoing equations are valid only for uniformly heated plates, cylinders, and spheres. This is because of the fact that Eqs. (6) and (7) contain, implicitly, solutions to the thermal conduction problems in only the bodies specified. It has been found, however, that this does not detract from their usefulness in estimating times of peak thermal stress in related configurations.

Reference

[1] Timoshenko, S. and Goodier, J. N., *Theory of Elasticity* (McGraw-Hill Book Co., New York, 1951), 2nd ed., pp. 408–409.

Technical Comment

Erratum: "Correlations for Theoretical Rocket Thrust with Shifting Expansion"

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IN the 18th line of the second column of the above engineering note, the words "The foregoing" should read "Above."

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